Recitation Session 5

24.10.2018

Note: You need Adobe Reader to be able to see the animations in this pdf.

1 Saddle-Node Bifurcation

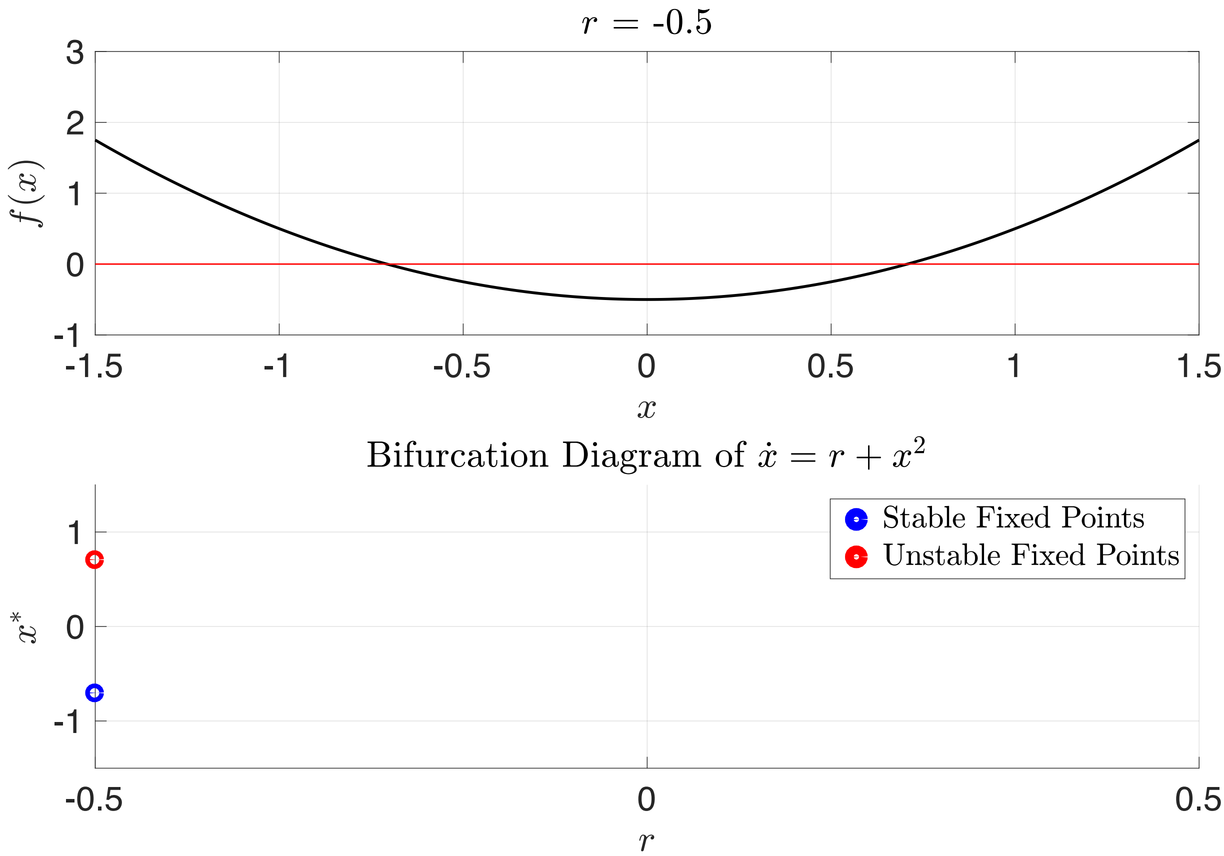


Figure 1: The figure on top shows how $f(x) = r + x^2$ varies as r is swept from -0.5 to 0.5. The lower figure plots the corresponding bifurcation diagram. Both animations show a snapshot for each value of the bifurcation parameter r.

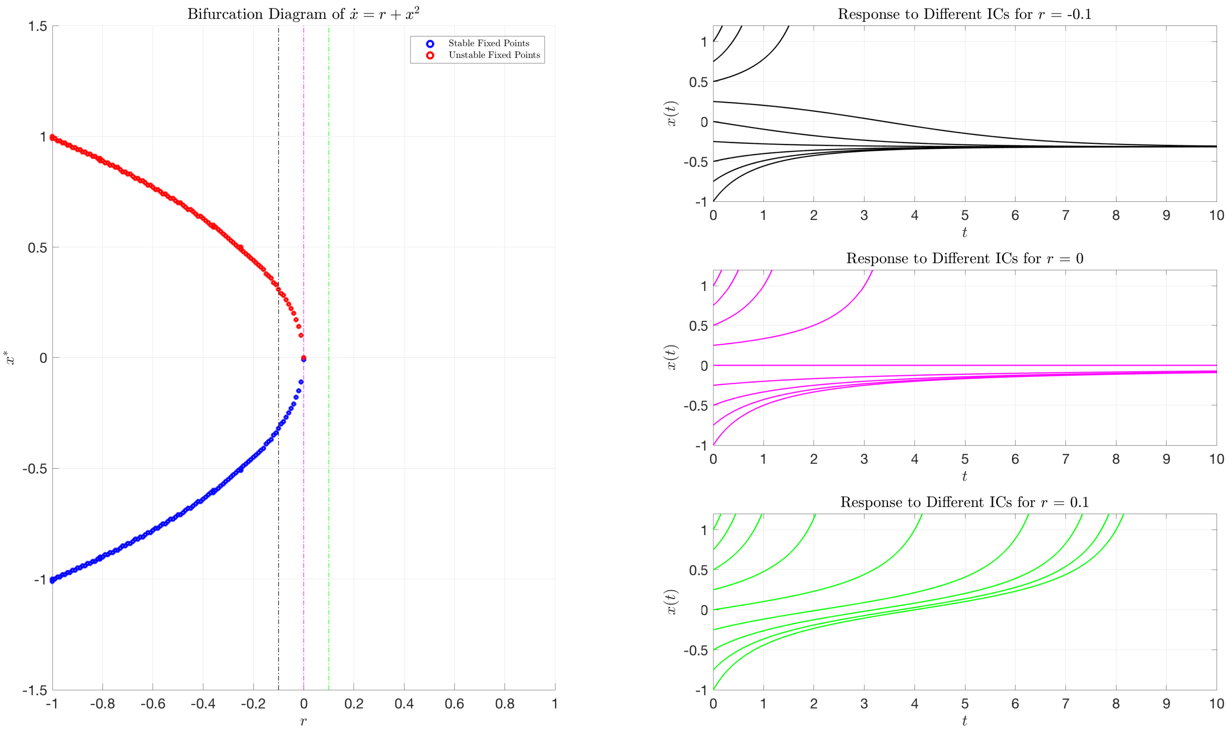


Figure 2: The figure to the left shows the bifurcation diagram for $\dot{x} = r + x^2$. Three particular values for the bifurcation parameter r are chosen $r = -0.1, 0,$ and 0.1. The dynamical system is simulated starting with multiple initial conditions. The temporal response is thus plotted for each of the particular values of r.

2 Transcritical Bifurcation

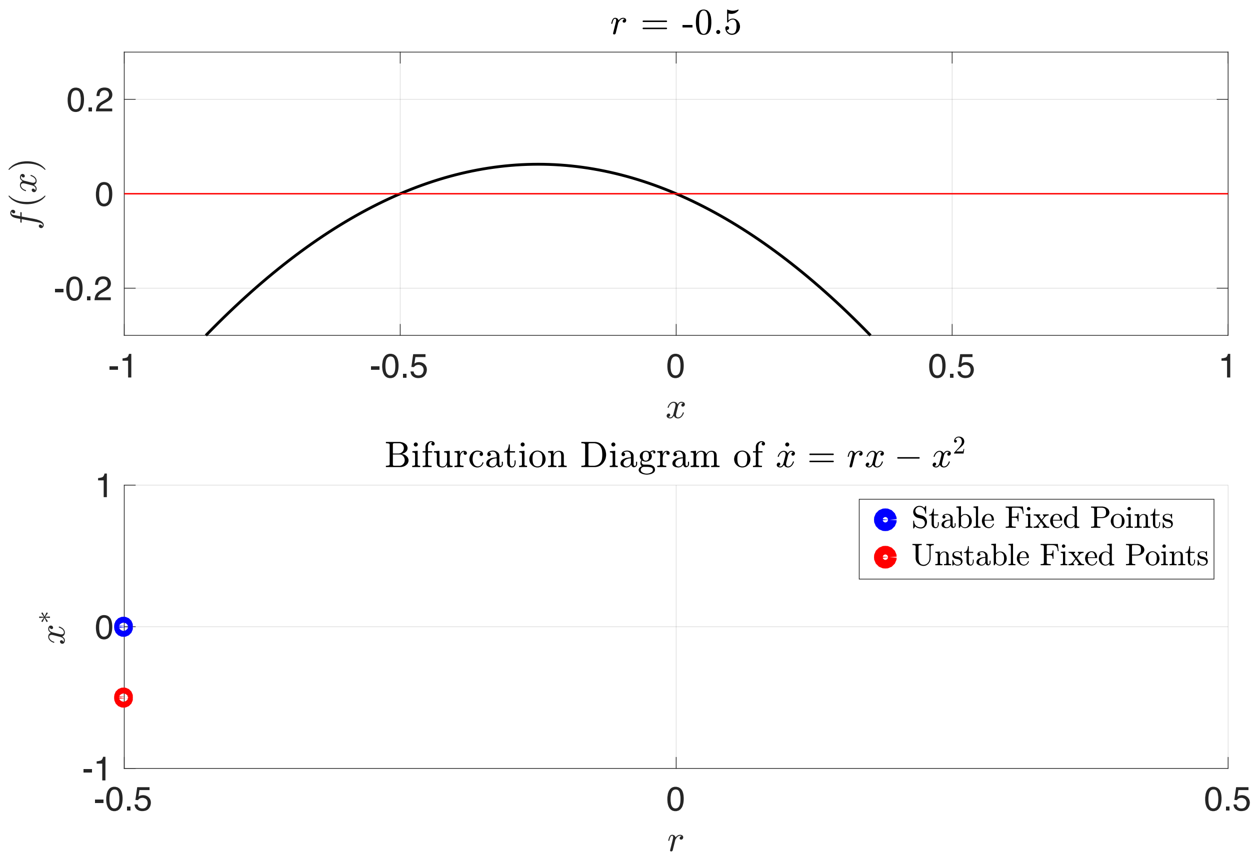


Figure 3: The figure on top shows how $f(x) = rx - x^2$ varies as r is swept from -0.5 to 0.5. The lower figure plots the corresponding bifurcation diagram. Both animations show a snapshot for each value of the bifurcation parameter r.

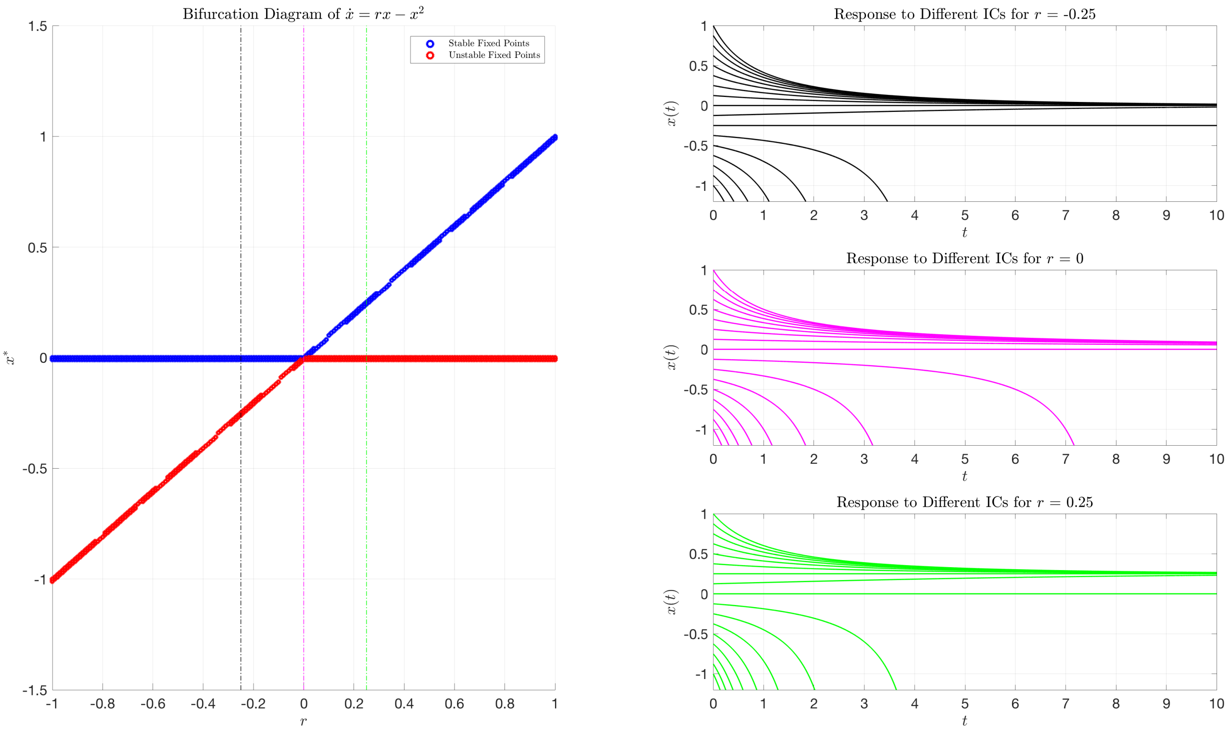


Figure 4: The figure to the left shows the bifurcation diagram for $\dot{x} = rx - x^2$. Three particular values for the bifurcation parameter r are chosen $r = -0.25, 0$, and 0.25. The dynamical system is simulated starting with multiple initial conditions. The temporal response is thus plotted for each of the particular values of r.

3 Pitchfork Bifurcation

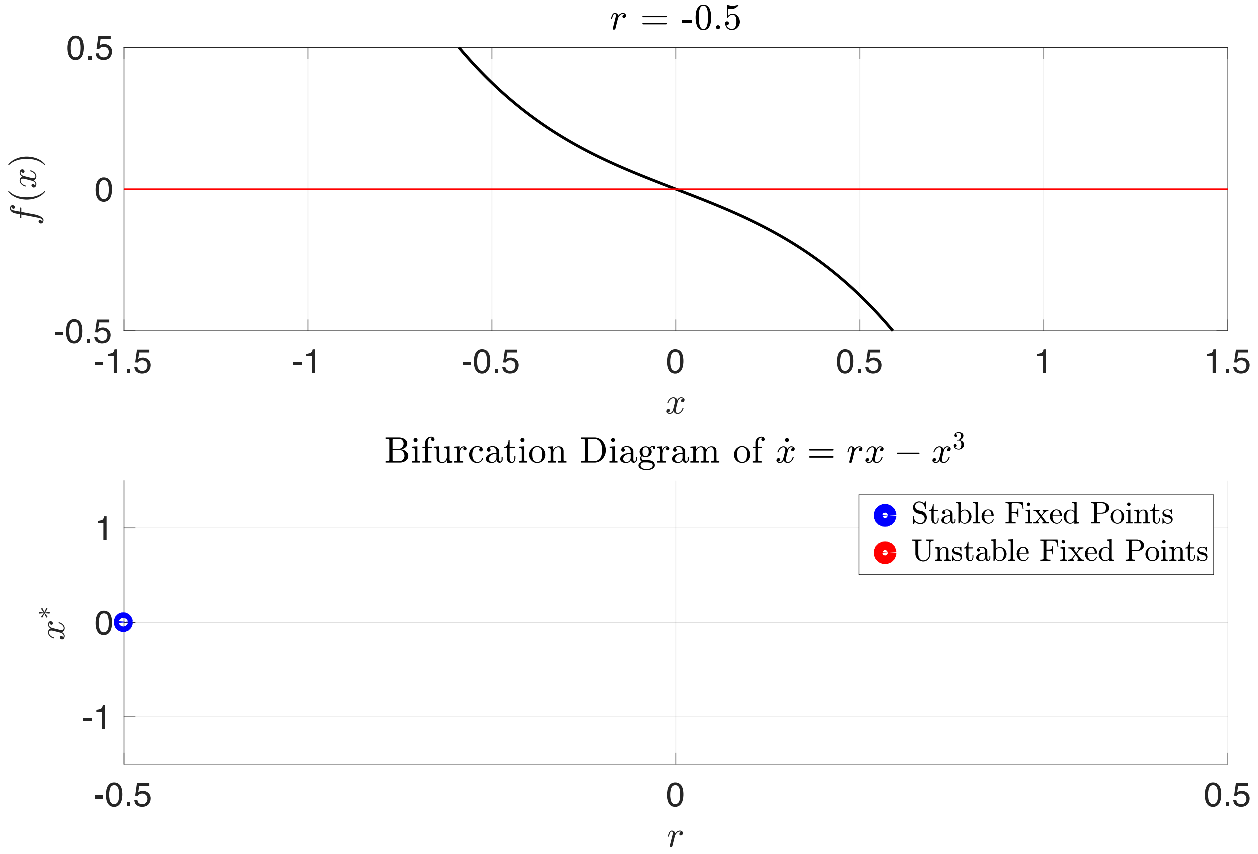


Figure 5: The figure on top shows how $f(x) = rx - x^3$ varies as r is swept from -0.5 to 0.5. The lower figure plots the corresponding bifurcation diagram. Both animations show a snapshot for each value of the bifurcation parameter r.

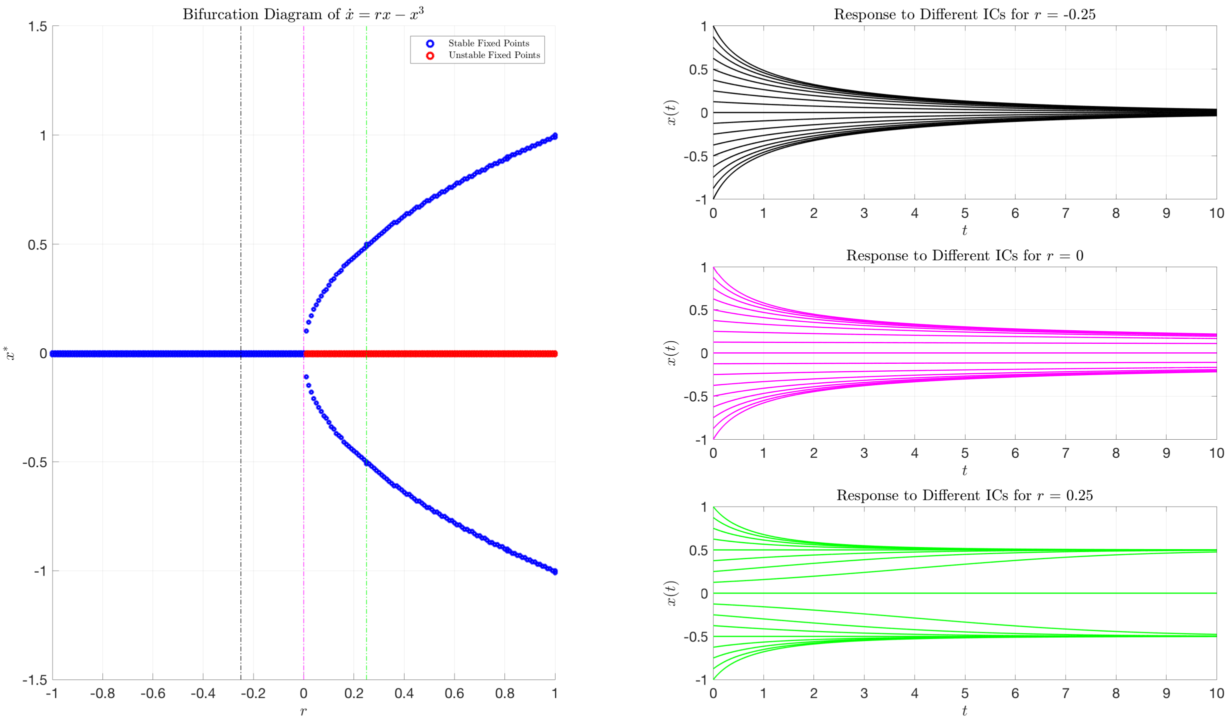
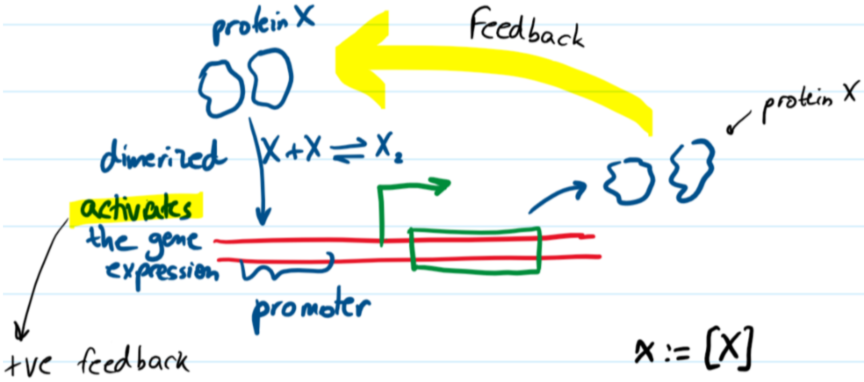


Figure 6: The figure to the left shows the bifurcation diagram for $\dot{x} = rx - x^3$. Three particular values for the bifurcation parameter r are chosen $r = -0.25, 0$, and 0.25. The dynamical system is simulated starting with multiple initial conditions. The temporal response is thus plotted for each of the particular values of r.

4 Biological Example: Gene Expression with Positive Feedback

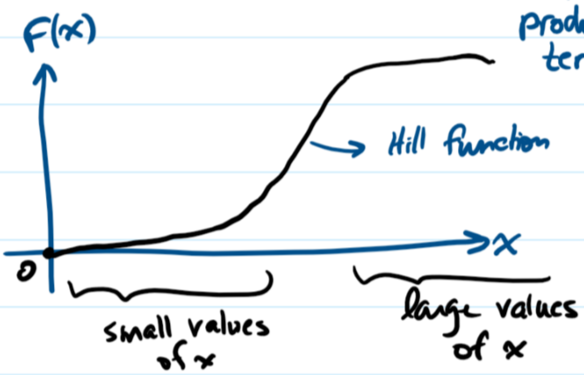
Gene Expression with positive Feedback:



$$x := [X]$$

degradation rate

"rate" of evolving x : $\dot{x} = \underbrace{F(x)}_{\text{production term}} - \underbrace{\gamma x}_{\text{degradation term}}$



$$F(x) = \frac{x^{2}}{1 + x^{2}}$$ related to "dimerization"

$$\therefore \quad \dot{x} = \frac{x^2}{1+x^2} - \gamma x$$

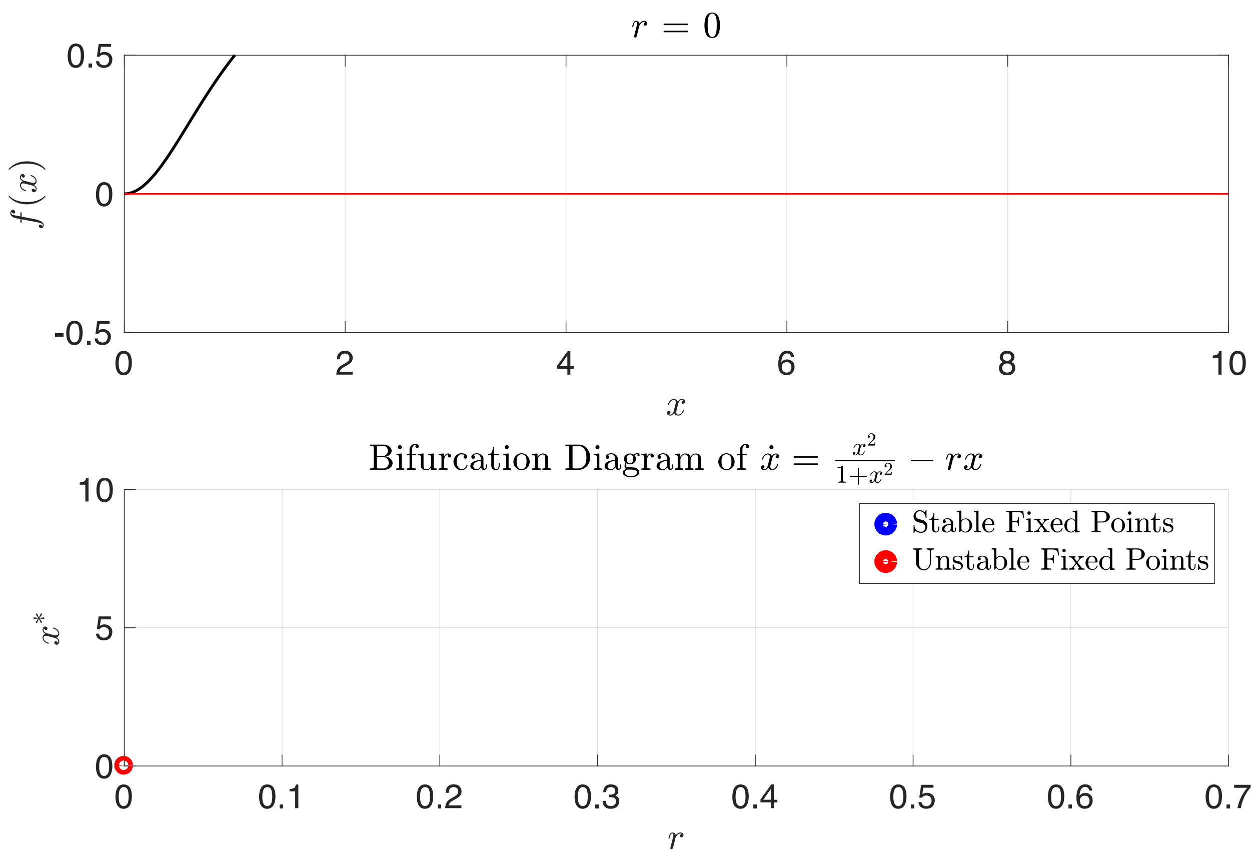


Figure 7: The figure on top shows how $f(x) = \frac{x^2}{1+x^2} - rx$ varies as r is swept from 0 to 0.7. The lower figure plots the corresponding bifurcation diagram. Both animations show a snapshot for each value of the bifurcation parameter r.

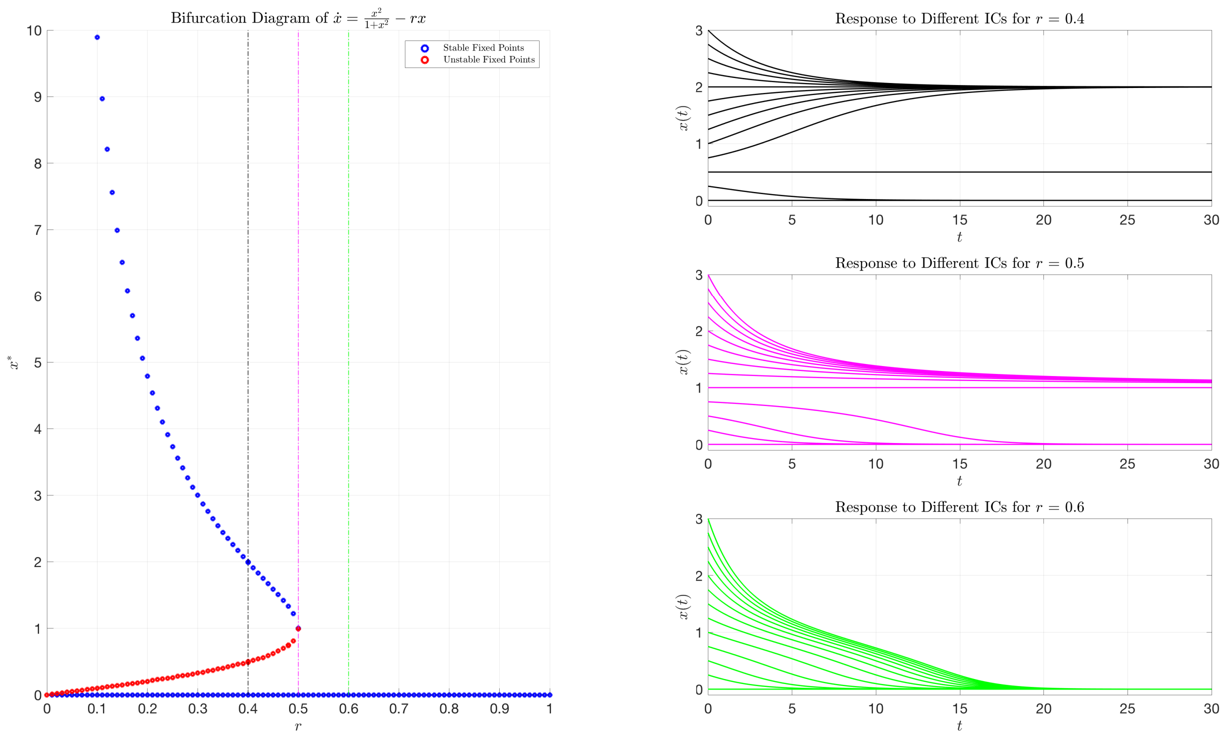


Figure 8: The figure to the left shows the bifurcation diagram for $\dot{x} = \frac{x^2}{1+x^2} - rx$. Three particular values for the bifurcation parameter r are chosen $r = 0.4, 0.5$, and 0.6. The dynamical system is simulated starting with multiple initial conditions. The temporal response is thus plotted for each of the particular values of r.